Bandit Multiclass Linear Classification: Efficient Algorithms for the Separable Case

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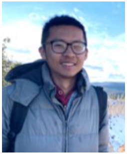

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(USC)

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(Microsoft)

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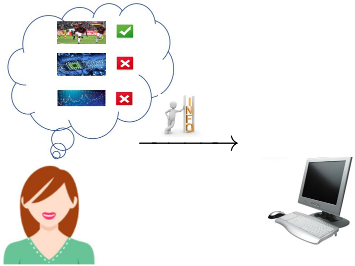

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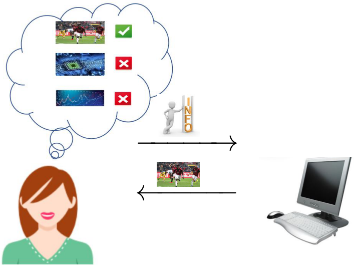

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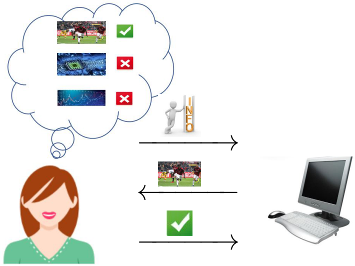

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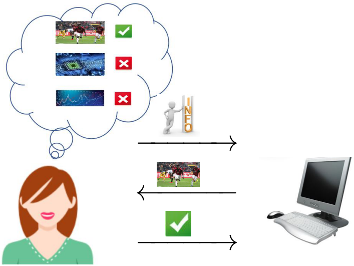

Goal: minimize the total number of mistakes $\sum_{t=1}^{T} z_t$.

Challenge: efficient algorithms in the separable setting

Definition

A dataset is called γ-linearly separable if there exists $w_1, \ldots, w_K$ such that

$$\langle w_y, x \rangle \geq \langle w_{y'}, x \rangle + \gamma, \qquad \forall y' \neq y,$$

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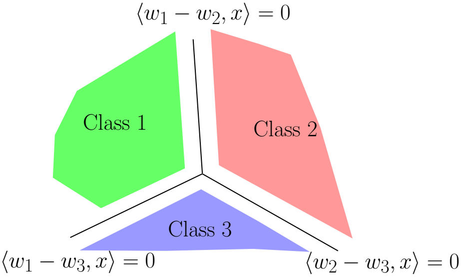

Related work

Algorithm	Mistake Bound	Efficient?

[1] See also [HK11, BOZ17, FKL$^+$18, ..] that have similar guarantees

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Contribution: first efficient algorithm that breaks the $\sqrt{T}$ barrier

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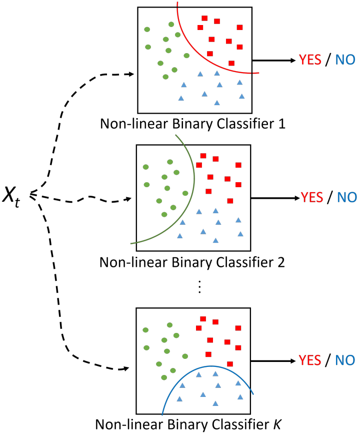

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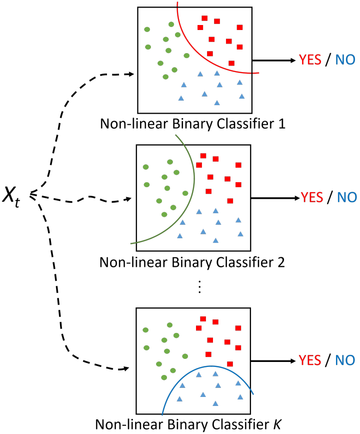

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$\widehat{y}_t \leftarrow$ any one of those YES labels

If all of them respond NO:
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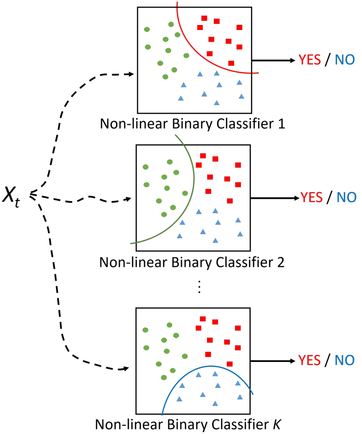

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$$\boxed{\mathbb{E}[\#\text{mistakes}(\text{alg})] \leq K \sum_i \#\text{mistakes}(i)}$$

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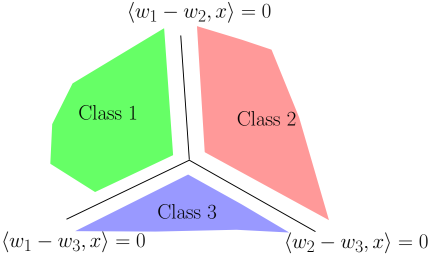
- Each non-linear binary classifier learns the support of class i, which lies in an intersection of $K-1$ halfspaces with a margin [KS04].



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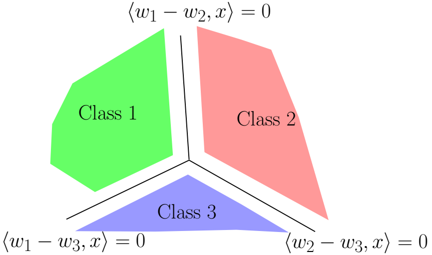

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$$K(x, x') = \frac{1}{1 - \frac{1}{2}\langle x, x'\rangle}.$$

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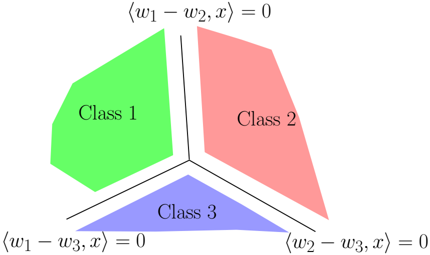

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- $\boxed{\text{Thu. Poster\#158}}$