

Statistical and Computational Foundations of Machine Learning
NYU Tandon School of Engineering
Spring 2023, Homework #0

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Submit your solutions by email to david.pal@nyu.edu with subject "Homework #0". Use LaTeX and submit your solution as a PDF file. Use the template provided on the course web page. Email your solutions by **23:59:59 EST, January 31, 2023**.

Collaboration is allowed on homework, but solutions must be written independently. If you use external sources (books, online documents), list them in your solutions. See homework policy on the course webpage for more details.

Question 1

Suppose n children, among them Alice and Bob, stand in a row in a random order.

- (a) (1 point) Let r be an integer between 0 and $n - 2$. What is the probability that there are exactly r children between Alice and Bob?
- (b) (1 point) What is the expected number of children between Alice and Bob?
Hint: $\sum_{k=1}^n k = k(k+1)/2$ and $\sum_{k=1}^n k^2 = n(n+1)(2n+1)/6$
- (c) (1 point) Let a_n be the expected number of children between Alice and Bob. Prove that

$$\lim_{n \rightarrow \infty} \frac{a_n}{n} = \frac{1}{3}.$$

Question 2

Let n, d be positive integers such that $n \gg d$. Let A, B, C, D be matrices of dimensions $d \times n, n \times d, d \times n, n \times 1$ respectively.

- (a) (1 point) Suppose we run the standard matrix multiplication algorithm to compute the products AB, BC, CD . Express the asymptotic time complexity of each of the three computations using big-O notation! Provide explanation for your answer.
- (b) (1 point) How do you compute the product $ABCD$? Describe your algorithm in detail. Try to come up with an algorithm with as low time complexity as you can. Express the time complexity of your algorithm using big-O notation!